

# ENERGY BALANCE MODELING APPLIED TO A COMPARISON OF WHITE AND GREEN ROOF COOLING EFFICIENCY

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## **Abstract**

We constructed a general energy balance model applicable to both green and non-green roof surfaces. It includes equations for shortwave and longwave radiation, convective, latent and conductive heat fluxes. For convection, we use a temperature gradient model with a wind-dependent coefficient. Latent heat flow for green roofs is simulated using the Bowen ratio. For the energy balance, we use an equilibrium model, ignoring the heat content changes within the roof layer itself. The major site-specific unknowns, that vary depending on the rooftop characteristics, are the convection coefficients and the Bowen ratio. In this study, we determined them using a hierarchical approach applied to field data for three control and three green roofs monitored at Penn State University. Close fits between the field data and the model were obtained that simultaneously constrained the unknowns. We also used a calibrated control roof model and data to answer a practical question for green roof research: How well do green roofs cool compared to white roofs? To answer this we asked: *What albedo is needed on a bright or white roof to reproduce the cooling observed on a green roof?* This was estimated by raising the albedo on the calibrated non-green model until it simulated the reduced temperatures observed on the green roofs. For the data collected during July 2003, our model required 'equivalent albedos' in the range 0.7–0.85. Such albedos are comparable to the highest available from white roof surfaces. The implications of this finding for green and white roof research are discussed.

## **Introduction**

In urban areas, building rooftops comprise a substantial fraction of the total land surface area, which means their physical properties are important determinants of the urban environment. Typically black impervious, traditional rooftops contribute directly to two ubiquitous urban environmental problems: the heat island effect and combined sewage-stormwater overflows (CSO's). For the building owner, dark roofs mean higher summertime energy consumption rates for cooling. Green rooftops, which have higher albedos and which retain water that is evaporated to the atmosphere, can mitigate these problems, especially if they are implemented on a wide scale.

A simple modeling methodology rooted in climatology – called 'energy balance' modeling - is available to study the role of roofs in the urban heat island and building energy consumption rates (1). Energy balance refers to the physical fact that energy cannot be created nor destroyed so that the solar and longwave radiation energy received by a rooftop layer during

any time interval must exactly equal, or 'balance,' the energy gained by that layer minus that lost from the layer during the same time interval. The physical equations that describe these gains and losses are widely used in climate studies (1). Green roof research is an ideal application for energy balance modeling.

### Energy Flux Terms For Rooftops

There are seven energy flux<sup>1</sup> terms that need to be included within the energy balance. These are: (i) shortwave radiation downwards; (ii) shortwave radiation reflected upwards; (iii) longwave radiation downwards; (iv) longwave radiation emitted upwards; (v) sensible heat loss or gain; (vi) latent heat loss and (vii) heat conduction downwards or upwards from the room below the roof. Equations for most of these flux terms are readily available from standard atmospheric science and heat transfer literature (1). The major exceptions to this are the sensible heat flux, for which an 'all purpose' formula applicable to any building surface does not exist, and, to a lesser extent, the latent heat flux for complex vegetated surfaces.

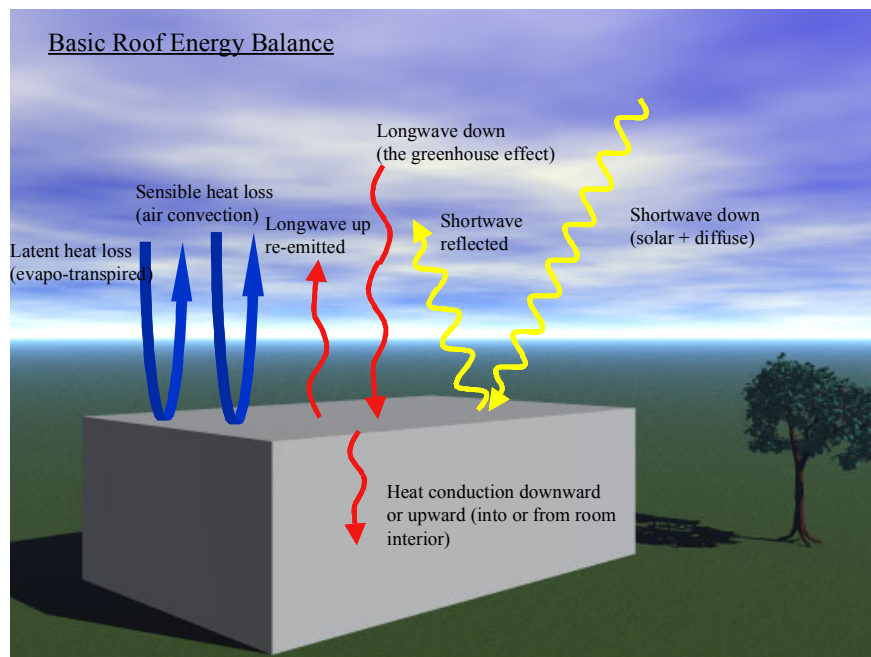


Figure 1: The seven energy fluxes<sup>1</sup> considered in the energy balance model.

Sensible heat transfer is a complex turbulence process involving winds, temperature gradients and boundary layer flows (2). As such it is not possible to generalize for the many different building geometries and environments that exist. However published urban climatology literature suggests the surface-to-air temperature gradient and wind speed are dominant factors (3, 4, 5, 6, 7). As such we adopt a formula with these two factors and with unknown coefficients that we determine using monitoring data on whatever roof is being studied. The equations for our model, including the sensible heat flux, are given in the appendix.

<sup>1</sup> "Flux" means energy passing through a unit area per unit time, such 'Watts / meter<sup>2</sup>' or 'BTU / foot<sup>2</sup> per hour', etc.

Latent heat flow in our model is assumed to be zero for non-green roofs corresponding to the assumption that during non-rainfall periods, water is generally not present on the roof. This assumption is most valid during warm seasons when rainfall quickly either drains or evaporates from the rooftop. For green roofs we adopt a standard approach used in climate models for land surfaces: latent heat flux is proportional to sensible heat flow with a proportionality coefficient known as the “Bowen ratio” (1, 8).

In this paper we will not be presenting applications using the Bowen ratio, but confine our studies to calibrating the model for the non-green roof and applying it to a comparison of the cooling power of white versus green surfaces. Results for our work on the Bowen ratio will be presented in another study.

The following is the statement of energy balance for the rooftop layer being monitored, expressed in terms of the seven energy fluxes and the heat capacity of the roof layer:

$$SW_{down} - SW_{up} + LW_{down} - LW_{up} - Q_{convection} - Q_{conduction} - Q_{latent} = C_{roof} \cdot \frac{d}{dt} \left[ \frac{T_{roof} + T_{ceiling}}{2} \right] \quad (1)$$

where SW, LW,  $Q_{sensible}$ ,  $Q_{conduction}$  and  $Q_{latent}$  refer to shortwave, longwave, sensible, conductive and latent heat transport, respectively.

The term on the right-hand-side of equation (1) is the rate of change of the heat content of the roof layer, which we calculate as the rate of change of the average roof layer temperature times a heat capacity coefficient for the roof layer unit area. To understand this equation, if the left-hand-side of (1) is positive, this means the roof layer is gaining more energy per unit time than it is losing. This gain in energy per unit time must appear as, and equal, the rate of increase of the heat content of the roof layer.

Non-green rooftops, in most cases, will be low mass systems to minimize structural load and, as such, cannot store much heat. This means they will quickly reach a temperature that will balance the heat and gain terms on the left hand side of the energy equation. Such systems are said to be in ‘quasi-equilibrium’ with the external forcing terms because the time delays are small between the actual temperatures observed on the roof and the equilibrium temperatures the roof would achieve if the forcing is held constant. In such cases equation (1) can be simplified by setting the right-hand-side to zero. The resulting equation is a non-linear equilibrium system, rather than a non-linear delayed system, which is computationally easier to work with.

Green roofs too will often be of low mass, but for buildings that have the loading capacity for intensive treatments and deeper soil mediums, an equilibrium assumption will not hold as well. This will increasingly be the case if relatively large volumes of water are present on the green roof since water adds significant mass and high heat capacity to the roof. In the first application presented in this paper we apply the model to a non-green roof low mass structure, located at Penn State, so that we can use the equilibrium version of energy balance with greater confidence. In our second application using green roof data, the effects of the time delays due to the roof’s greater mass are evident.

## Penn State Field Data

The Penn State Center for Green Roof Research (<http://hortweb.cas.psu.edu/research/greenroofcenter/>) has developed and instrumented a green roof field experiment in Central Pennsylvania. The experiment consists of 6 separate buildings, 3 with green roofs and 3 with control dark roofs (figure 2). Each of the roofs and buildings are extensively monitored for temperature, meteorological conditions and water retention and runoff. The waterproof membrane, drainage layer, growth medium and plants are identical for each green roof. The vegetation is *Sedum spurium*. Details on the building structures and roofs are given in reference 9.



Figure 2. Green and control roof field experiment at Penn State University (9).

Figure 3 shows averaged control rooftop surface temperatures and averaged green rooftop temperatures for the month of July 2003. The averages were made among all three buildings for both the control and green roofs. In addition, the data shown are hourly averages of temperature data taken every 5 minutes. For example, the noontime temperatures are the hourly average of 5-minute data collected between 11:00 am and noon. These data directly demonstrate the cooling potential of green roof surfaces compared to dark impervious surfaces. Peak temperatures can be 30° Celsius or more lower (equal to 54° Fahrenheit lower or 30° Kelvin lower– the energy balance model temperature scale) on the green rooftops.<sup>2</sup>

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<sup>2</sup> Temperature data in the energy balance model and in the figures are in degrees Kelvin. To convert degrees Kelvin to Centigrade, subtract 273.15.

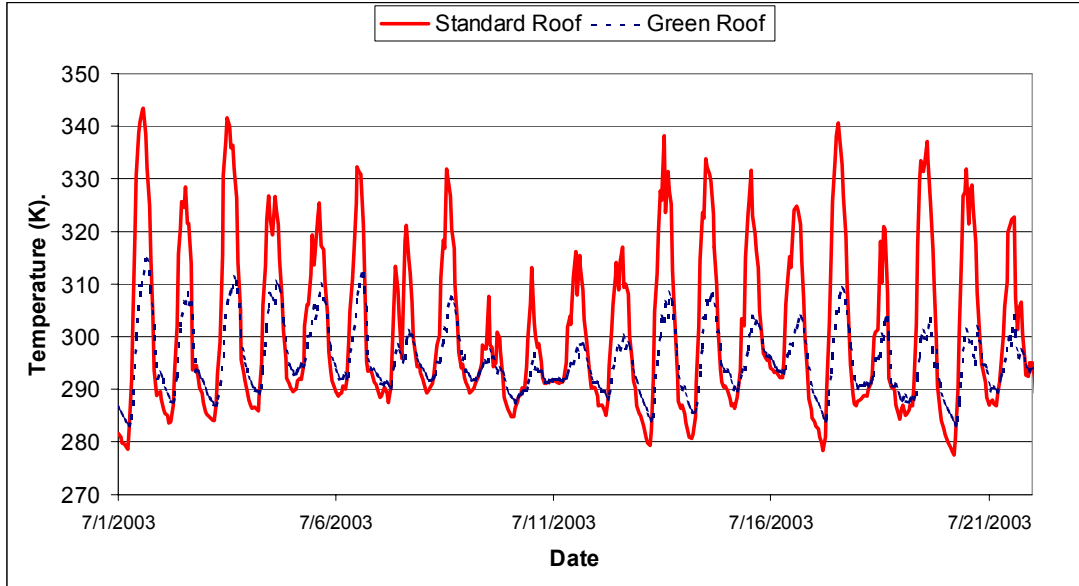


Figure 3: Average control and green rooftop surface temperatures observed on the Penn State University field experiment during July 2003 (9).

### **Model Simulations**

As a first application of the energy balance model, we performed a simulation of the control rooftop temperature data shown in figure 3. For this simulation we made reasonable estimates for the rooftop albedo and longwave emissivity based on their color and material composition. The thermal conductivity was known from the insulation material and dimension (9). Latent heat was assumed to be zero for the control roofs.

With the exception of the convection coefficients  $\gamma_1$  and  $\gamma_2$  in equation A5, all other model data were available either from the temperature data and meteorological recordings or else from standard published literature. Therefore we treated the convection coefficients as unknowns to be determined by optimizing the model agreement with the measured rooftop temperatures shown in figure 3. The optimization was made using the root-mean-square-error (RMSE) as the metric to be minimized. Figure 4 shows the best fit obtained by minimizing the RMSE.

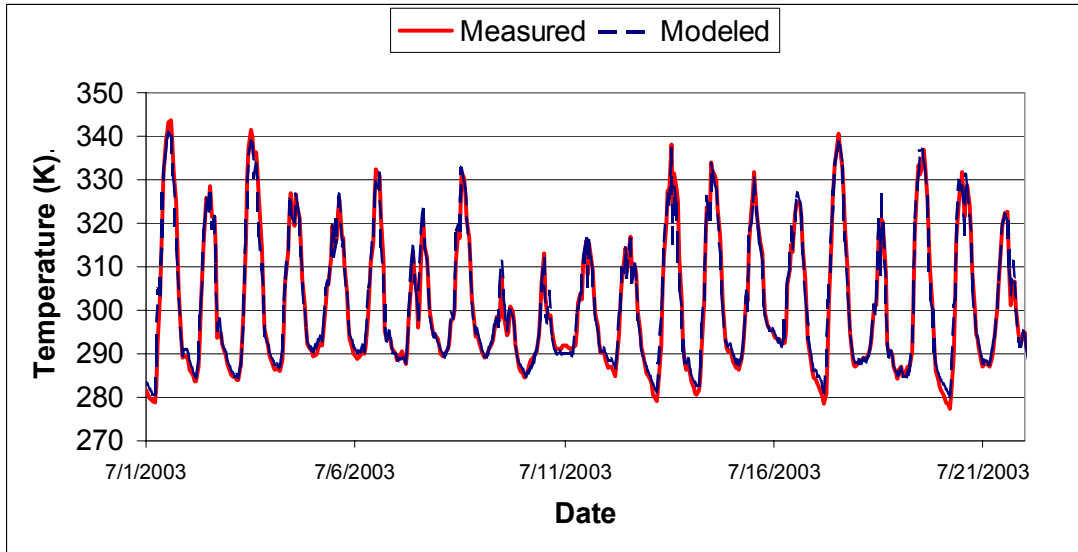


Figure 4: Modeled versus measured rooftop temperatures for July 2003 using hourly averages of 5-minute meteorological data, and adjusting the convection coefficients in equation A5 to minimize the RMSE measure of error.

Figure 4 shows that the model agreement is excellent. This fit was obtained with values for  $\gamma_1$  and  $\gamma_2$  of 6.6 and 10.3, respectively. These values, combined with the varying wind speed data, imply total convection coefficients in front of the  $(T_{\text{roof}} - T_{\text{air}})$  term in equation A5 ranging from 10-22  $\text{W/m}^2\text{-K}$ . The available literature on such coefficients is sparse. However our range agrees well with that reported by Berdhal and Bretz (3) in their study of rooftop convection, in which they find a range of 18-25  $\text{W/m}^2\text{-K}$ . Figure 4 uses hourly averaged data for the solar forcing. If instantaneous data were used, the model shows short-term variability against the measurements, which we interpret as meaning the rooftop temperatures better reflect the average solar gain over the prior hour rather than the instantaneous gain, due to thermal inertia effects.

### **The 'Equivalent Albedo' of Green Roofs Compared to White Roofs**

With the control roof model thus calibrated, we pose a practical question for green roof research: *what albedo would be required on a non-green roof to reproduce the surface temperatures observed on the green roofs, as shown in figure 3?* We refer to this albedo as the 'equivalent albedo' of green roofs. It combines in a single number both the latent and shortwave reflective cooling channels operating on green roofs (figure 1). It offers a simple and illuminating way of comparing the cooling efficiency of green roofs against white roofs. Before showing the results, we remark that if the equivalent albedo of green roofs is low compared to the albedo of white roofs, this could argue favorably for adopting white roofs. Alternatively if the equivalent albedo is high compared to that achievable on white roofs this could argue favorably for green roof adoption.

To answer this question, we use the identical model parameters and meteorological forcing used for the results in figure 4 but with the green roof surface temperatures as the target data and the albedo treated as an unknown to be determined by 'optimizing' the agreement.

We found that the model temperature cycles temporally preceded the green roof temperature cycles by approximately one hour. We interpret this as the effect of the increased mass and heat capacity of the green roof and retained water as compared to the control roofs. As a result 'optimizing' the fit in this case is not a straightforward matter of minimizing the RMSE because of the time lag between the simulated and observed temperature cycles. Nevertheless, we minimized the RMSE metric, along with visual inspection, and found that a range of 'equivalent albedos' gave reasonably good fits that bracket the data. The agreement varied over the month.

Figure 5 shows the model output with the range of albedo's that our model needed to bracket the data for the month of July. The model data shown in figure 5 are time lagged by one hour for visual clarity. We estimate the bracketing equivalent albedo range to be between 0.7 and 0.85. Earlier in the month the equivalent albedo of 0.7 matched the temperature peaks well while later in the month, this albedo made the model too hot. For the latter periods, an albedo of 0.85 cooled the model peaks in better agreement with the data. The physical reason for this variability likely has to do with the water status of the green roofs over the month. If more water is present at certain times, we expect the latent heat cooling to increase and thus elevate the equivalent albedo of the roof. We will be testing this hypothesis in a follow-on study.

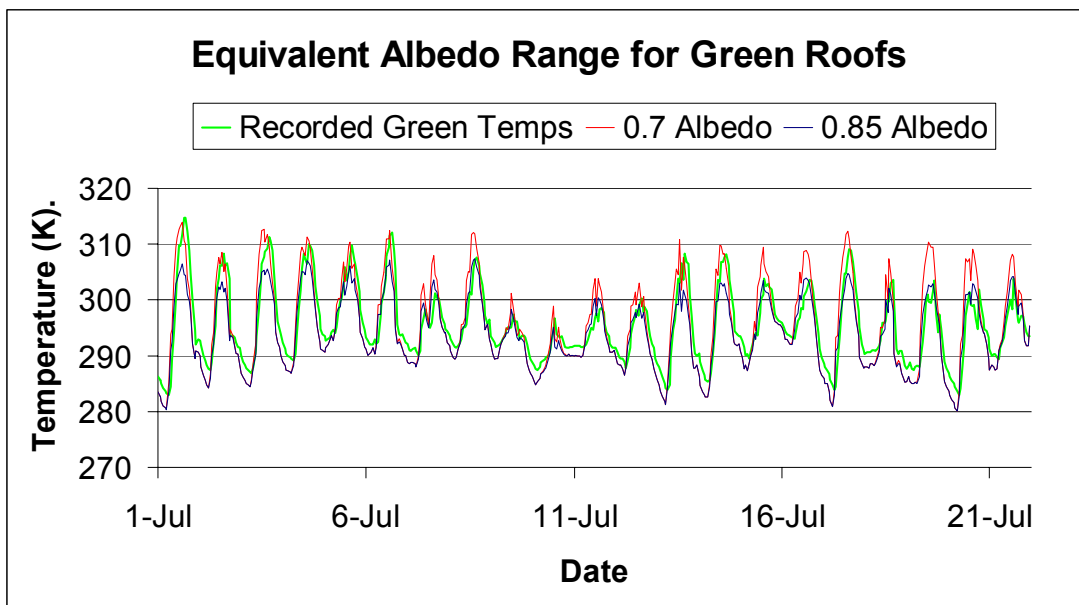


Figure 5: Simulation of the green roof surface temperatures using only a raised albedo on the calibrated control roof model. A range of albedos (0.7-0.85) was needed to bracket the data.

## **Discussion**

The equivalent albedos found for the Penn State green roofs have implications for green roof research. White or bright roofs are a competing technology to green roofs as a method to reduce the urban heat island and rooftop temperatures. The chief advantage of a white roof is that it is relatively inexpensive to install. But a key question that needs to be addressed is whether white roofs cool more effectively than green roofs? If white roofs do, this would be a

second important advantage. The equivalent albedos found in this paper provide an answer to this question.

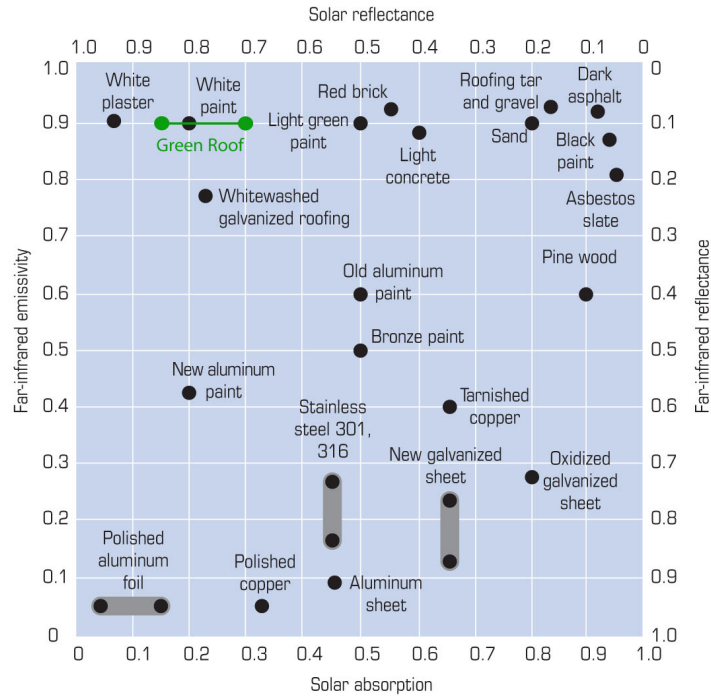
Surface solar reflectivities in the range of 0.7 – 0.85 are among the brightest surfaces available from white coatings. Figure 6 shows reflectivity (albedo) and long wave emissivity data for a number of standard building and rooftop materials (black dots) (10). The albedo values are plotted on the horizontal axis and the corresponding emissivities are plotted on the vertical axis. It is seen that white paint typically averages an albedo of 0.8. However maintaining high albedos on white surfaces is difficult. Without regular washing, the albedos of white surfaces rapidly drop due to natural weathering and soiling (11). Albedo declines for white surfaces in outdoor environments have been reported to decline by an average of 0.15 over a year. In some cases larger declines happen within two months (11). While the brightness can be restored by regular washing, the burden and costs of doing this one or more times a year will likely be a major deterrent to most building owners.

Green roofs by comparison are not cooling primarily through albedo, but through latent heat loss. Our equivalent albedo experiments suggest that they are cooling as effectively as the brightest possible white roofs, but without the need for washing like white roofs. There are of course other maintenance costs for green roofs, but for purposes of cooling, the maintenance may not be as intensive and burdensome. Moreover, green roof maintenance potentially will be an inviting and pleasurable activity for many building owners. In comparison, very bright white roofs with high glare are harsh environments visually.

Although green roofs are cooling through latent heat (along with a higher natural albedo compared to black roofs), our equivalent albedo experiment allows us to place green roof markers on figure 5 for comparison to other materials. Vegetated surfaces have longwave emissivities in the range of 0.9 or higher (1), also among the highest possible for most materials. Combined with an equivalent albedo of 0.7-0.85, green roofs are operating in the desirable upper left corner of the albedo-emissivity chart. For this data set at least, only one material (white plaster, not a rooftop candidate) has greater surface cooling properties. This finding suggests that vegetation are optimized by nature for cooling efficiency.

In this study we have used the green roof *surface* temperatures as the target data to be simulated by the equivalent albedos. With respect to the *heat flow* downwards into the room below, however, a more appropriate target temperature would be the temperature observed at the conventional rooftop level *below* the green roof layer. It is this subsurface temperature which governs the heat flow into the room below, as given by equation A6. This subsurface temperature was monitored at the Penn State experiment and is significantly lower than the green roof surface temperature, simply because it is insulated by the green layer above it. Using this subsurface temperature as the target for our equivalent albedo experiment would therefore require albedos even higher than the 0.85 needed for the surface temperature. Therefore the case can be made that green roofs are reducing heat flows into the building below to levels not achievable by white roofs.





Source: Florida Solar Energy Center

Figure 6: Data on solar reflectivity (horizontal axis) and infra-red emissivity (vertical axis) for a number of common building materials (10). This study of green roof equivalent albedos allows us to hypothetically depict where green roofs lie in comparison to such materials, with respect to cooling performance.

Although green roofs are more expensive to install than white roofs, the many other benefits of green roofs could make them more desirable and cost-effective (12). These other benefits include: stormwater runoff mitigation, roof service lifetime extension, building amenity value, biodiversity value, and direct air quality benefits. White roofs, by comparison, only offer the one surface cooling benefit but this will require burdensome maintenance to be fully realized. In the same way that natural vegetated surfaces maintain themselves, a properly functioning green roof could be self-perpetuating with respect to cooling.

The 'equivalent albedo' concept is a practical metric that should be promoted in green roof and urban heat island research. To our knowledge it has not been introduced heretofore. This may be because its utility becomes clear only when comparing, as we do, the two fundamental urban heat island strategies: vegetation and albedo increases. We encourage future energy balance studies to replicate the findings we report. We have only studied one field site, in one climatological regime, for an extensive green roof, with particular vegetation and during a limited time period. The varying drought and water status of other sites, climates, vegetation and annual time periods, need to be considered and studied as well. The cooling efficiency of intensive versus extensive green roofs is another useful application for energy balance modeling.

## Appendix

### Model Equations

Following are the equations we adopt for the seven energy fluxes depicted in figure 1:

$$Shortwave_{down} = Direct\_Solar + Diffuse\_Solar \quad (A1)$$

$$Shortwave_{up} = \alpha \cdot Shortwave_{down} \quad (A2)$$

$$Longwave_{down} = (0.605 + 0.048 \cdot e^{1/2}) \cdot \sigma \cdot T_{air}^4 \quad (A3)$$

$$Longwave_{up} = \varepsilon_s \cdot \sigma \cdot T_{roof}^4 \quad (A4)$$

$$Q_{sensible} = \gamma_1 \cdot u^{0.8} (T_{roof} - T_{air}) \quad \text{if } u > 1.75, \text{ else } \quad Q_{convection} = \gamma_2 \cdot (T_{roof} - T_{air}) \quad (A5)$$

$$Q_{conduction} = \kappa (T_{roof} - T_{ceiling}) \quad (A6)$$

$$Q_{latent} = \begin{cases} 0 & \text{for non-green roofs (e.g. standard, white)} \\ \frac{Q_{convection}}{\beta} & \text{for green roofs} \end{cases} \quad (A7)$$

Following are brief definitions for the symbols:  $\alpha$  = albedo,  $\sigma$  = Stefan Boltzmann constant ( $5.67 \cdot 10^{-8}$  Watts/meter<sup>2</sup>-K<sup>4</sup>),  $T_{air}$  = ambient external air temperature,  $e$  = water vapor pressure of the atmosphere,  $\varepsilon_s$  = longwave emissivity of the rooftop,  $T_{roof}$  = rooftop surface temperature,  $u$  = windspeed,  $\kappa$  = thermal conductivity of the roof layer (Watts/m<sup>2</sup>-°K),  $T_{ceiling}$  = interior ceiling temperature inside room below roof,  $\beta$  = Bowen ratio of sensible and latent heat flux. All scientific units are metric in the model, meaning temperatures are measured in degrees Kelvin, windspeed is in meters/second, radiation and heat flow terms are in Watts/meters<sup>2</sup>, and vapor pressure is in millibars

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